WHY LONG TERM DYNAMIC ELASTICITIES DIFFER FROM CROSS-SECTION ELASTICITIES ?

Jean-Loup MADRE
Department Economy and Sociology of Transport INRETS

François GARDES
University Paris I

ABSTRACT

Data are often lacking for a good estimation of models:
- either agregate time-series are available (e.g. from National Accounts), thus the heterogeneity between individuals cannot be taken into account,
- or a survey gives a detailed description of one point in time, but no proper information on long term dynamics.

What can be derived in terms of dynamic elasticities, from the cross-sectional analysis of heterogeneous behaviors shown through only one survey ?

Three examples can be taken according to socio-demographic variables: age, residential location and income.

The recent literature on relative income effects and social interactions can be related to the old problem of the difference between cross-section and time-series estimation. On the one hand, the distribution of consumption can be estimated from the same survey by comparing individuals who, ceteris paribus, are at different positions in the income distribution, in their residential location or life cycle. On the other hand, change in expenditure due to income changes, to moving home or to ageing can be measured for the same agent (or the same type of agent) between two periods, thus using individual time-series (or pseudo-panel) data.

1. INTRODUCTION

What can be derived in terms of either long or short run dynamic relationship (often measured in term of elasticity), from the cross-sectional analysis of heterogeneous behaviour shown through only one survey ? What explanation can be given for this discrepancy ?

Three examples are taken according to age (section 2), residential location (section 3) and income (section 4); in sections 2 and 3, the metropolitan region of Grenoble is taken as case study. For instance, the cross-sectional income elasticity of an expenditure or a tax is a good indicator of its redistributive effect: regressive for a negative value, progressive for a value higher than 1, with references 0 or 1 for neutrality. However, it is misleading to use it as a dynamic long term elasticity.

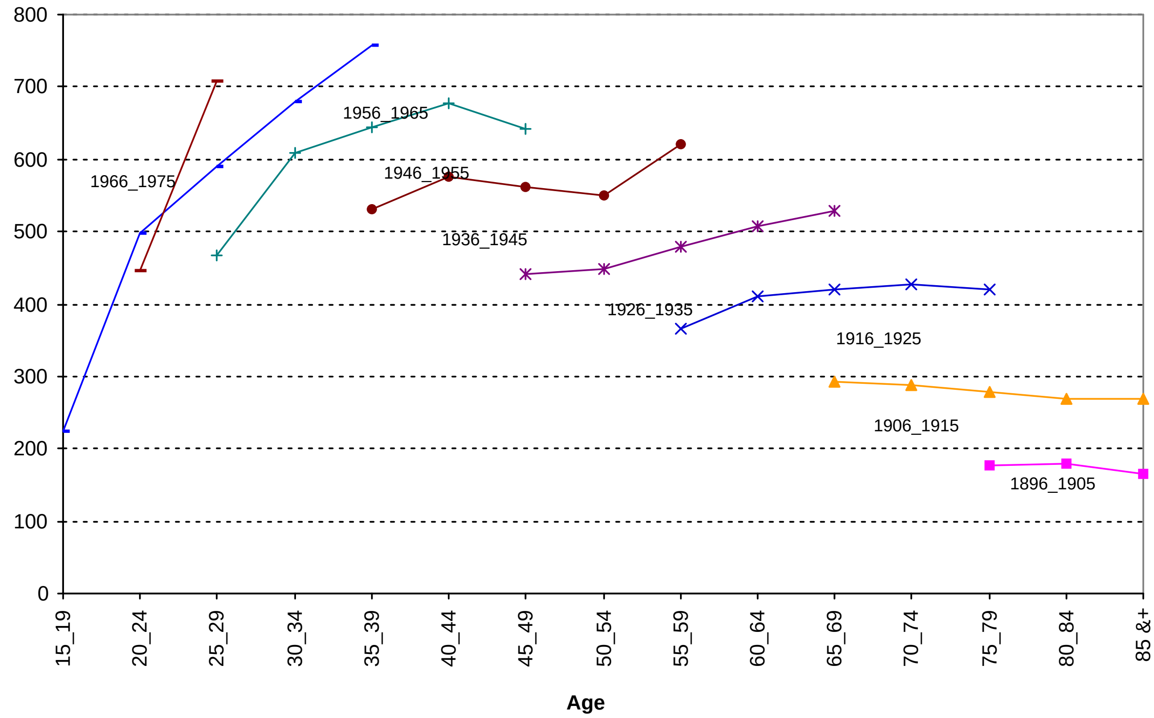
"Statistical relationships established from cross-sectional data do not provide a 'track' along which responses move over time. Model coefficients estimated from cross-sectional data about the effects of differences in levels of a causal factor are not the same as those established from panel data about the effects of changes in the level of the same factor" [Goodwin, 1997].

The fundamental assumption inherent in a model estimated on a cross-sectional data set (a cross-sectional model) is that a behavioral measure at time t, Y(t), can be expressed as a function of explanatory variables, X(t), and an error term, e(t), also at time t when the data were taken. Systematic variations in Y are often related to variations in X within the same cross section. This is especially the case when The unexplained variance represent a large proportion of the variance, as it is generally for individual data. Applying this relation to forecasting would involve longitudinal extrapolation of cross-sectional variations; changes in behavior over time would be predicted based on differences in behavior across individuals.Despite the fact that most forecasting models in use are cross sectional and embody this assumption, its validity has not been tested in any rigorous manner... There are several conditions that must be satisfied for this assumption to hold :

1) behavioral changes are instantaneous,
2) behavioral changes are symmetric, or reversible, and
3) behavioral relation is stationary (invariant over time)." [Kitamura, 1990].

2. AGE: LIFE CYCLE AND GENERATION EFFECTS

Figure 1. Evolution of individual motorisation in Grenoble by cohort



Sources: INSEE Census in 1975, 1982, 1990 and 1999.

The changes in motorisation for subsequent cohorts along a 15 years portion of their life cycle is studied in Madre et Bussière, 2002 and Dargay et al., 2000. The same Figure is drawn for the Metropolitan Region of Grenoble for a 24 years period from 1975 to 1999 census.

From the cross-sectional distribution observed in 1980, we could derive that:
- elderly people should abandon their car quite early (at least after retirement),
- the patronage of public transport would develop with the ageing of population.

More than twenty years later, we have observed through the follow-up of cohorts that:
- elderly people keep their car till the age when they cannot drive any more (generaly beyound 75),
- thus, the proportion of elderly among public transport users has decreazed dramaticaly, especially for rail modes, in which the driver cannot help passengers.

In fact, the age effect depicted in cross-section is not a pure life cycle effect, but mainly a generation effect: cohorts born before World War II in Western Europe or in Japan have had low access to private car.

On the example of concentric zones in the Metropolitan Region of Grenoble, Table 1 shows that maintaining a fixed distribution of motorisation according to the age of household gives unrealistic forecasts: car ownership would decrease because of population ageing, and it should be worse if we had taken the distribution observed in 1975 instead of 1990 [Bussière et Madre, 1998]. Considering the data from 1999, it appears that the Age-Cohort model estimated on 1975 to 1990 data overestimate car ownership in 2010, except for outer suburbs.

Table 1. Motorisation forecasts in the Metropolitan Region of Grenoble

	Grenoble	Inner suburbs	Outer suburbs	All
% of households without a car :				
1975	39	24	33	32
1982	34	18	23	25
1990	33	16	15	21
1999	31	15	11	19
Projection 2010 (1)	27	10	8	14
2010 fixed by age(2)	33	20	19	23
% of multi-car households :				
1975	9	16	15	13
1982	12	23	26	20
1990	15	33	40	29
1999	15	34	48	33
Projection 2010 (1)	19	42	58	42
2010 fixed by age (2)	16	31	39	30

Source: INSEE Population census 1975 to 1999 and calculations by INRETS.
(1) from Age-Cohort modelling based on 1975 to 1990 census.
(2) Motorisation by age of the head of household fixed at its 1990 value.

3. RESIDENTIAL LOCATION : DOES THE COMPARISON BETWEEN CITY CENTER AND SUBURBS ALLOW AN ESTIMATION OF THE IMPACT OF URBAN SPRAWL ?

The effects of urban sprawl cannot be derived from the comparison between households living in city centers, inner or outer suburbs. Table 2 shows how households the head of which lived already in the same zone of the Metropolitan Region of Grenoble at the date of the previous census differ from those who lived elsewhere.

Table 2. Old and new households in the Metropolitan Region of Grenoble

	Grenoble			Inner suburbs			Outer suburbs			All		
	82	90	99	82	90	99	82	90	99	82	90	99
% of households :												
- Old inhabitants	73	70	61	74	75	73	76	73	75	74	73	70
- New settlers	27	30	39	26	25	27	24	27	25	26	27	30
All	100	100	100	100	100	100	100	100	100	100	100	100
- Gone for other zones	15	20	13	12	12	14	6	5	12	11	12	13
Average age of the head of households :												
- Old inhabitants	53	55	56	49	51	54	52	53	54	51	53	54
- New settlers	35	34	33	37	38	38	40	40	41	37	37	37
All	48	49	47	46	48	50	49	49	51	47	49	49
- Gone for other zones	40	41	42	38	39	40	37	38	36	39	39	40
% of households without a car :												
- Old inhabitants	36	34	30	20	17	15	27	18	13	27	22	18
- New settlers	28	29	33	14	13	15	9	7	5	18	17	20
All	34	33	31	18	16	15	23	15	11	25	21	19
- Gone for other zones	11	9	8	14	12	11	22	17	20	14	11	13
% of multi-car households :												
- Old inhabitants	12	16	17	24	33	35	23	37	45	19	29	34
- New settlers	11	14	13	22	32	32	37	47	55	22	29	31
All	12	15	15	23	33	34	26	40	48	20	29	33
- Gone for other zones	30	41	48	28	37	37	14	25	25	27	31	37
Number of adults for 100 households :												
- Old inhabitants	183	178	170	215	209	197	209	207	200	202	198	191
- New settlers	168	161	150	188	183	170	204	194	189	185	178	167
All	179	173	162	208	203	190	208	203	197	198	193	184
- Gone for other zones	199	192	186	193	184	173	167	168	158	192	189	172
Number of cars per 100 adults :												
- Old inhabitants	42	46	51	48	56	61	46	57	66	46	54	61
- New settlers	49	53	53	57	65	69	63	72	79	56	63	66
All	44	48	52	51	58	63	50	61	69	48	56	62
- Gone for other zones	60	69	75	59	68	73	55	64	66	59	63	72

Source: INSEE 1982, 1990 and 1999 census.

N.B.

- "old inhabitants" are the households whose head lived already in the same zone (outer suburb, inner suburb, city of Grenoble) at the date of the previous census (1975, 1982 or 1990),
- "new settlers" the other households (i.e. those whose head lived elsewhere),
- "gone for another zone" the households whose head lived in this zone at the previous census and have moved for another zone of the Metropolitan Region of Grenoble; from the 1982 and 1990 census, we have no information on those who have left this Metropolitan Region.

Generally speaking, the "new settlers" are younger than the "old inhabitants". The new inhabitants have more cars, although there are slightly less adults in their households. This difference for motorisation is lower in the central city than in the suburbs, and lower in inner than in outer suburbs. Indeed, the rural population of outer suburbs have a traditional behaviour, although the new inhabitants have often kept their job in central zones, as shows the analysis on work location.

New settlers have an average commuting distance 30% higher than old inhabitants of the central city and of inner suburbs, and 65% higher in the outer suburbs (Table 3). Those who have moved farther from the city center (e.g. from inner to outer suburbs) have commuting distances twice higher than those who have settled nearer to the city center. It means that travel behaviour can be considered neither symmetric nor reversible

Table 3. Commuting distances for the inhabitants of the Metropolitan Region of Grenoble in 1999

	Zone of residence in 1999			
	Grenoble	Inner suburbs	Outer suburbs	All
Zone of residence in 1990:				
Grenoble	5.9	7.1	12.8	6.8
Inner suburbains	3.4	6.8	15.0	7.3
Outer suburbs	6.2	8.7	9.9	9.5
Out of the Metropolitan Region	10.6	10.6	18.9	13.1
Old Inhabitants	5.9	6.8	9.9	7.6
New Settlers	7.8	9.1	16.4	11.0
All	6.6	7.4	11.9	8.7

Source: INSEE 1999 census; distance in kms.
N.B.:
- "old inhabitants" are the households whose head lived already in the same zone (outer suburb, inner suburb, city of Grenoble) at the date of the previous census (1975, 1982 or 1990),
- "new settlers" the other households (i.e. those whose head lived elsewhere).

4. CROSS-SECTIONAL INCOME ELASTICITIES DIFFER SUBSTANTIALLY FROM LONG TERM DYNAMIC ELASTICITIES

The recent literature on relative income effects and social interactions can be related to the old problem of the difference between cross-section and time-series estimation. The social distribution of consumption can be estimated by comparing individuals who, ceteris paribus, are at different positions in the income distribution (i.e. have different relative incomes) in the same survey. On the contrary, the change in expenditure due to income changes can be measured for the same agent (or the same type of agent) between two periods, thus using individual time-series (or pseudo-panel) data. Any discrepancy between the estimated income elasticities in cross-section and time-series means that similar agents (with respect to all characteristics except income) with different relative incomes are not identical, as the income position generates relative income effects which are due either to social interactions (as supposed by Duesenberry, 1949), or to latent variables related to the income position (for instance parents' characteristics or liquidity constraints, which are not observed in family expenditure surveys).

Such differences between cross-section and time-series estimates of demand functions have been observed in recent empirical work: for instance, Gardes et al. [2002] analyse the bias in income and total expenditure food elasticities estimated on panel or pseudo-panel data caused by measurement error and unobserved heterogeneity. Our results suggest that unobserved heterogeneity implies a downward bias to cross-section estimates of income elasticities of at-home food expenditure and an upward bias to estimates of income elasticities of away-from-home food expenditure. Moreover, the magnitude of the differences in elasticity estimates across methods of estimation is roughly similar in U.S. and Polish expenditure data : for instance, despite some differences between the estimations, the relative income elasticity of food at home is around 0.2 based on a number of different methods with PSID (Panel Survey on Income Dynamics) data from US (1984-1987), while the time-series estimates (within or first differences) are 0.4. A Hausman test strongly rejects the equality of these cross-section and time-series estimates. In a Polish panel (1987-1990), the total expenditure elasticities for at-home food are much larger than those based on PSID data. Higher elasticities are to be expected for a country in which food's share in total expenditures is three times higher than in the U.S. Cross-section elasticities are estimated to be around 0.5, while the time-series estimate is 0.8. On the contrary, the cross-section elasticity for food away from home is estimated to be around 1 in the U.S. while the time-series elasticity is around 0.4 (similar results are obtained for Poland, although the estimates are less accurate, due to the absence of food away from home for almost all Polish households during this period). Similar results have been obtained on pseudo-panels of French and Canadian surveys [Cardoso et al., 1996a and 1996b; Gardes et al., 1996].

This research has shown that endogeneity biases exist in the cross-section estimates for half of the commodities. For example, the cross-section income effect, is significantly greater for most expenditures on services, while changes in expenditure on housing over time are more strongly related to income changes than are the differences between two households in the same survey.

4.1. Specification

The following reduced form Engel curve is estimated:

$$w_{it} = a_i + b_{i1}\ln(Y_t/a(p_t)) + Z_t.c_i + e_{it} \quad (1)$$

where w_{it} is the budget share of good i, $Y_t/a(p_t)$ household's real total expenditure with $a(p_t)$ a Stone price index, Z_t household's characteristics, e_{it} a stochastic term which captures measurement errors and unobserved preferences. We estimate equation (1) taking into account possible measurement errors in total expenditures using its predicted value obtained from instrumentation equation with disposable household income and a few socio-demographic characteristics.

4.2. The data

The French family budget surveys are conducted every five years, the last one in 2001. They detail the pattern of private expenditures of about one thousand households. Classic problems affect the data : errors of measurement, particularly for the income variables, systematic differences with aggregate data (for instance for tobacco), change of the relative prices between the waves (over almost one year), etc. We use four surveys: 1979, 1984, 1989 and 1995.

The individual data are aggregated according to five cohorts, two education levels and two locations (Paris, other). Only 4 over the 80 cells have less than 60 households and 10 less than 100 (representing less than 1.3% of the population). The average cells size is 539.

4.3. Results

Cross-section and time-series income elasticities have also been estimated on the Polish panel and for the Canadian Family Expenditures surveys (Table 4).

Table 4. Cross-section and time-series income elasticities for transport expenditures

	France Pseudo-panel of 1979, 84, 89 surveys		Polish panel (all transport expenditures)		Canada	
	Between	Within	Between	Within	Between	Within
Purchase of vehicles	1.233	1.366			1.419 (0.046)	1.209 (0.107)
Expenditures for personal transport	1.006	0.999	1.72 (0.05)	1.17 (0.06)	1.161 (0.036)	1.203 (0.038)
Public transport	2.541	1.587			1.061 (0.071)	0.881 (0.083)
Income Elasticity Of the virtual price:						
Purchase of vehicles	+0.2					-0.35
Personal transport expenditures	0		-0.9			+0.07
Public transport	-1.2					-0.41
Sources: France: Cardoso-Gardes, 1996a Canada: Gardes et al., 1996 Poland: Gardes, 2004						

It appears that :

1) In the cross-section dimension, transport services are generally luxury expenditures. The ratio of cross-section elasticities between services and the corresponding durable is around 2.
2) Cross-section public transport elasticities are greater than the time-series, contrary to the durables. Allthough, personal transport expenditures are similar to the time-series.
3) Within elasticities of services seem to be greater for the poor than for the rich [Cardoso and Gardes, 1996b].

4.4. The income elasticity of the complete prices: the case of transport expenditures

Substitution exists between consumption activities which can be obtained, either by acquiring directly market services, or purchasing goods in order to transform them by domestic production. Such is the case with food at home, consumed through domestic production using time and other on monetary inputs, compared to restaurant services, or transport needs, which can be met by walking, driving, etc. A wage rate increase, also increases the opportunity cost for time spent on any domestic activity, which elevates the complete price of consumption activities requiring time, such as food at home. This implies a negative effect of income on this domestic activity, facilitating the substitution towards market services.

This effect appears only if the monetary price of the market substitute does not increase with the household income: thus, the effect disappears whenever the income change concerns all households, for instance when the wage rate increases by the same amount, between two periods, for the whole population. On the contrary, the substitution between food at home and food away is likely when comparing the rich and the poor in a cross-section. Thus, the difference between cross-section and time-series income effects provides some information on the relative income effect.

The difference between cross-section and time-series consumption laws can be interpreted by means of virtual prices (see Appendix). The price elasticities for transport expenditures are calibrated as the half of within income elasticities. The virtual price for transport is clearly increasing for greater income for personal transport and decreasing for public transport. This may be related to the cost of time used during the transport: this cost is smaller for public transport, as some activity can be made or rest can be obtained during this type of transport. On the contrary, personal transport needs for the driver to use fully his (or her) time. Richer households have greater opportunity costs for time, so that the component of the virtual price related to time use is increasing more rapidly with relative income for public transport.

5. CONCLUSIONS

Using dynamic specification of the consumption model with age cohort instrumentation seems to be an efficient method to obtain plausible estimates of long and short term elasticities based on cross-section data. The obtained long term income elasticities for transport expenditures are significantly greater than short term ones, both for estimations using individual data or aggregate time-series. This is very important for income effects calibration. When computed on cross-section the under-estimation of expenditure elasticities can be at least 50%. Consequently, the computed dynamic parameters of consumption function increase significantly the role of income effects in the decomposition analysis of the inter-temporal change in budget shares between goods and services when compared with the static frame.

Time-series unbiased estimates of income effects are generally smaller than cross-section for transport expenditure, especially for public transport. It could explain why traffic forecasts are often over-estimated, when the effect of economic growth is derived from cross-sectional calculations. This difference is probably due to the time constraint which is modelled by virtual prices and which appears as significant in three very dissimilar countries such as France, Canada and Poland. It will have to be checked by pairing time use surveys and family expenditure surveys. Such a pairing will be performed on recent INSEE and Canadian data.

In the cross-section dimension, transport services are generally luxury expenditures. The ratio of cross-section elasticities between services and the corresponding durable is around 2. Cross-section public transport elasticities are greater than the time-series, contrary to the durables. On the contrary, personal transport expenditures are similar to the time-series. Within elasticities of services seem to be greater for the poor than for the rich.

Last, let us consider the necessary conditions given by Kitamura [1990] for considering cross-sectional relationships as longitudinal ones:

1) Behavioral changes are very seldom instantaneous in a sector, where investments are important (e.g. car purchase or infrastructure building).
2) We have given examples of assymmetry in section 2 for households moving from center to periphery and vice versa.
3) Elasticities are not invariant over time (e.g. income-elasticity of car ownership is decreasing [Dargay et al., 2000]); that's why flexible specifications have to be used for the estimation of long term relationships.

REFERENCES

Banks, J., Blundell, R. and Lewbel, A., 1997, Quadratic Engel curves and consumer demand, The Review of Economics and Statistics, n° 79, pp. 527-539.

Bettendorf, L., Barten, A.P., 1995, Rationnement dans les Systèmes de demande: Calcul des Prix Virtuels, Economie et Prévision, n° 121, 101-108.

Blundell, R., Robin, M, 1999, An Iterated Least Squares Estimator for Conditionally Linear Equations Models, Journal of Applied Econometrics n° 14, pp. 209-232.

Bussière Y., Madre J.-L., 1998, Démographie et effets de génération dans la prospective de la demande de transport urbain à long terme : les cas de Grenoble et de Puebla à l'horizon 2010, Association de Science Régionale de Langue Française, Puebla.

Cardoso, N. and Gardes, F., 1996a, Estimation de lois de consommation sur un pseudo-panel d'enquêtes de l'INSEE (1979, 1984, 1989), Economie et Prévision n° 5, pp. 111-125.

Cardoso, N., Gardes, F., 1996b, La consommation des ménages pauvres, Revue Economique.

Dargay J., Madre J.-L., Berri A., 2000, Car ownership dynamics seen through the follow-up of cohorts : a comparison of France and the UK, Transportation Research Record n° 1733, pp. 31-38.

Deaton A., 1986, Panel Data from a Time Series of Cross Sections, Journal of Econometrics, vol. 30, pp. 109-126.

Duesenberry, J.S., 1949, Income, Saving and the Theory of Consumer Behavior, Harvard University Press, Cambridge, MA.

Gardes, F., 2003, Riemannian Consumers, w.p. Crest.

Gardes, F., Duncan, G., Gaubert, P., Starzec, C., 2005, A Comparison of Consumption Laws estimated on American and Polish Panel and Pseudo-Panel Data, Journal of Business and Economic Statistics (forthcoming).

Gardes, F., Langlois, S. and Richaudeau, D., 1996, Cross-section versus time-series income elasticities, Economics Letters n° 51, pp. 169-175.

Gardes, F., Starzec, C., 2002, Evidence on Addiction Effects from Households Expenditure Surveys: the Case of the Polish Panel, Econometric Society European Meeting, Venice, August.

Goodwin Ph., 1997, Have panel surveys told us anything new ?, in Panel for Transportation Planning, Methods and Applications. Ed. Golob, T., Kitamura, R. and Long, L (chap. 3).

Gurgand, M., Gardes, F. and Bolduc, D., 1997, Heteroskedasticity in Pseudo-Panel, working paper, Lamia, Université Paris I.

Kitamura, R., 1990, Panel analysis in transportation planning: an overview,. Transportation Research, 24A (6), pp. 401-415.

Madre J.-L., Bussière Y. (Eds), 2002, Démographie et Transport : Villes du Nord et Villes du Sud, l'Harmattan, 477 p., avril.

Mundlak, Y., 1978, On the Pooling of Time Series and Cross Section Data, Econometrica 46, 483-509.

ACKNOWLEDGEMENT

This research is conducted for the COST Action 355 WATCH "Changing behaviour for a more sustainable mobility", especially for Working Group 2 "Automobile and Multimodality". We are grateful to the French Agency for Environment and Energy Savings (ADEME), which has funded this research as PREDIT project.

Appendix: Measuring Shadow Prices

Suppose that the monetary price p_m and a shadow price π corresponding to non-monetary resources and to constraints faced by the households are combined together into a complete price. Expressed in logarithmic form, we have: $p_c = p_m + \pi$.

Consider two estimations of the same equation:

$$x_{iht} = Z_{ht}.\beta_i + p_{cht}.\gamma_i + u_{iht} \qquad (1)$$

for good i (i = 1 to n), individual h (h = 1 to H) in period t (t = 1 to T), with $Z_{ht} = (Z_{1ht}, Z_{2ht})$. These estimations are carried out on cross-section and time-series data using the same data-set.

Set $u_{iht} = \alpha_{ih} + \varepsilon_{iht}$, where α_{ih} is the specific effect which contains all of the permanent components of the residual for individual h and good i. As discussed by Mundlak (1970), the cross-section estimates can be biased by a correlation between the explanatory variables Z_{ht} and the specific effect. This can result from latent permanent variables (such as an event during childhood, parents' characteristics, or permanent wealth) which are related to some of the explanatory cross-section variables Z_{ht}: for instance, the relative income position of the household can be related to its wealth or its genetic inheritance. Thus, the correlation δ_i between the time average of the vector of the explanatory variables, $Z_{ht} = (z^k_{1ht})_{k=1 \text{ to } K1}$, transformed by the Between matrix:

$$BZ_{ht} = \{(1/T)\ \Sigma_t z^k_{ht}\}_{k=1 \text{ to } K1},$$

and the specific effect α_{ih} , $\alpha_{ih} = BZ_{ht}.\delta_i + \eta_{ih}$, will be added to the parameter β_i of these variables in the time average estimation: $Bx_{iht} = BZ_{ht}.(\beta_i + \delta_i) + \eta_{ih} + B\varepsilon_{iht}$, so that the between estimates are biased. The difference between the cross-section and the time-series estimates amounts to δ_i.

Let us now assume that the shadow price π_{iht} of good i for household h in period t, depends on variables Z_{1ht}, which also appear in the consumption function for good i:

$$x_{iht} = g_i(p_{ht}, Z_{ht}, S_{ht}) + u_{iht}$$

with p_{ht} the vector of prices p_{jht} containing (if it exists) a shadow, unknown component π_{jht}, and S_{ht} the vector of all other determinants.

We now assume that only the monetary component of prices change over time (the shadow component being related to permanent variables), while the different agents observed in the cross-section survey are characterized by different non-observed shadow prices (corresponding to individual non-monetary resources and constraints). Equation (1) writes on time-series (for instance in first differences between periods):

$$x_{iht} = Z_{ht}.\beta_i + p_{miht}.\gamma_i + u_{iht}$$

while on cross-section it is, supposing the price effect γ_i and monetary prices are the same on both dimensions:

$$x_{iht} = Z_{ht}.\beta'_i + u'_{iht} = Z_{ht}.\beta_i + \pi_{iht}.\gamma_i + u_{iht}$$

with obvious notations. Thus, the difference between the two estimations is:

$$Z_{ht}.\delta_{1i} = Z_{ht}.\theta_1\gamma'_i + (S_{ht}.\theta_2 + \lambda_{ih} + \mu_{iht}).\ \gamma_i$$

which allows to calculate the set of parameters θ_1 after calibrating the price effect measured by γ_i.

The marginal propensity to consume with respect to Z_{1ht}, when considering the effect of the shadow prices π_{jht} on consumption, can be written as:

$$dx_{iht}/dZ_{ht}= dg_i/dZ_{ht} + \Sigma_j\ (dg_i/d\pi_{jht}).(d\pi_{jht}/dZ_{ht}).$$

The second term will differ between cross-section and time-series because of the correlation of the shadow price with the endogenous variables Z_{ht}. So, comparing two different households surveyed in the same period, this bias adds to the direct unbiased consumption propensity with respect to Z_{ht}, as estimated on time-series data. For instance, the influence of the household head's age cohort or income may differ in cross-section and time-series estimations if the shadow prices depend on cohort effects or on the relative income position of the agent (note that the same effect may occur with respect to monetary prices).

The term $\Sigma_j\ dg_i/d\pi_{jht}.d\pi_{jht}/dZ_{ht}$ above can be used to reveal the variation of shadow prices over Z_{1ht}, $d\pi_{jht}/dZ_{ht}$, since it can be computed by resolving a system of n linear equations after having independently estimated the price marginal propensities $dg_i/d\pi_j = \gamma_{ij}$. We can also consider only the direct effect of the variables Z_{ht} through the price of good i, $\gamma_{ii}.d\pi_i/dZ$, so that:

$$d\pi_i/dZ = [\beta_i^{(c.s.)} - \beta_i^{(t.s.)}]/\gamma_{ii}. \qquad (2)$$

The price effect γ_{ii} is supposed to be the same for monetary and shadow prices. Thus, the change in the shadow price between two periods can be written as: $d\ln\pi_{iht} = \sum_k (d\pi_i/dz^k).dz^k_{ht}$. Under homogeneity (of degree m) of shadow prices over variables Z_{1ht}, the shadow prices can be computed as $\ln\pi_{ih} = m\sum_k(d\pi_i/dz^k).z^k_{ht}$. However, this homogeneity assumption is quite strong, and we will prefer to compute only the change in the log shadow prices[1].

[1] This model is presented more thoroughly and applied to rationality tests in Diaye *et al*. (2001).

The income elasticities of the shadow prices of food at home and food away from home expenditures are computed in Table 1 for the PSID and the Polish panel (using equation 2, and assuming that direct price elasticities are minus one half of the corresponding income elasticities). These estimated parameters are remarkably similar in both countries: positive and smaller than one for food at home, so that the full price of food at home is greater for richer than for poorer households. One interpretation is that rich people are time constrained and have a larger opportunity cost for the additional time spent on food at home compared to food away. This difference may be thought to be greater in the USA than in Poland. On the contrary, the income elasticity of the full price for food away from home is negative, and of the same magnitude in both countries.

In our analysis, the relation between cross-section and time-series estimates, modelled by shadow prices, is supposed to be linear over all of the distribution. It is however likely that the derivatives $d\pi_{jht}/dZ_{1ht}$ also depend on individual characteristics. This is for instance the case in Barten's model (1964) where relative prices depend on the family structure. This local dependency requires a geometric characterization of the consumer space.